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A bar of stub steel, of the same dimensions, was found to have a magnetic moment of 8,750,000. This variety of steel is used for the common grade of tools, and is comparatively soft. The magnetic moment of a bar of ordinary cast nickel was then obtained. Both bars were highly tempered to render the conditions of comparison as nearly equal as possible.

With the same formula the magnetic moment of a bar of cast nickel was found to be only 36,330; that is, about one two-hundred-and-twentieth as much as the moment of Fleitman's nickel.

These numbers are, of course, not perfectly exact, for the changes of magnetism in bars from time to time preclude perfect exactness; still the relation of the magnetic moments may be considered very accurate.

This result is certainly surprising; that the addition of $\frac{1}{8}$ per cent of magnesium to a bar of nickel should increase the magnetism 220 times shows that change of structure in a metal increases its magnetic capacity.

No. XXIV.—THERMAL CONDUCTIVITY OF GLASS AND SAND.

By C. B. Penrose.

Presented June 9, 1880.

In determining the conductivity of glass I used the same method that Forbes employed in determining the conductivity of iron.

A bar of glass is maintained, at one end, at a constant temperature. When the bar has reached a permanent state of heat, that is, when the amount of heat received by any portion exactly equals the amount given out by that portion, the temperature of a number of points on the bar are determined. The bar is taken so long that the heat at the heated end will not be sufficient to raise the temperature of the other end above that of the air. The temperatures determined are laid off as ordinates, the abscissas being the corresponding lengths of the bar. The equation of the curve thus formed can be determined. Then an exactly similar bar to the preceding is heated to a known temperature, and as it cools the temperatures are taken every minute, and thus the loss of temperature per minute is determined.

These diminutions of temperature, each corresponding to a known temperature of the bar, are laid off as ordinates of a second curve, each ordinate corresponding to an ordinate of the first curve, which expresses the temperature of the bar at any point. Thus every ordinate of the second curve will give the loss of temperature per minute, from that portion of the bar which is at the temperature given by the corresponding ordinate of the first curve. And the area of a definite portion of the second curve multiplied by the specific heat of the glass gives the amount of heat lost from a corresponding definite portion of the bar, in unit of time.

If we have a vertical section of any substance, of thickness x, one face at the constant temperature t_0 and the other at the temperature t, and $k =$ the conductivity of the substance, the quantity of heat that passes through an area A in unit time $= Q$, where

$$Q = K . A \frac{t_0 - t}{x}. \qquad \therefore K = \frac{Q}{A} \frac{1}{\frac{t_0 - t}{x}}.$$

If the section is an infinitely thin lamina of thickness $d\,x$, and if $d\,t =$ the difference of temperature of the two sides, —

$$K = \frac{Q}{A} \frac{1}{\frac{d\,t}{d\,x}} =, \text{ at the limit, } \frac{Q}{A} \frac{1}{D_x t}.$$

$D_x t$ can be found for any point of the bar, by finding the tangent at the corresponding point of the first curve. Hence if $A =$ area of cross-section of bar, $S =$ the specific heat of the glass, and "area" $=$ the area of the second curve beyond the point in question, the conductivity at that point is $K = \frac{S}{A} \frac{\text{area}}{D_x t}$.

The area of the second curve beyond the point in question, multiplied by S, evidently equals the whole amount of heat that passes, in unit time, through the section of the bar at that point; for the end of the bar is at the same temperature as the air.

I used a bar of flint glass, about 30 cm. long and 1.1 cm. in diameter. One end was inserted in the side of a metallic vessel, and heated to 100° by boiling water. The other end was supported on a piece of wood. Two screens of card-board were placed in front of the can and lamp to prevent the bar being heated by radiation. Seven points were marked on the bar: the first, 3 centimeters from the can, and the others, 1, 2, 3, 5, 7, and 9 centimeters from the first point. The temperatures were measured by a thermopile made by a junction of copper and iron wires, fastened like two links of a chain, so that they could be placed astraddle the bar, and thus make a very close connection.

After the water had boiled one hour the bar reached a permanent state. I then took the following observations of the temperatures of the different points, as shown by the deflections of the galvanometer. The temperatures are expressed in centimeters of the galvanometer scale, and consequently give the excess of the temperatures above that of the air.

	1st pt.	2d pt.	3d pt.	4th pt.	5th pt.	6th pt.	7th pt.
	7.2	4.9	2.9	2.3	1.4	1.0	0.4
	7.2	4.9	2.9	2.2	1.4	1.0	0.4
	8 2	4.9	3.2	2.3	1.7	1.1	0.4
	7.9	4.7	2.9	2.0	1.4	0.9	0.4
	6.9	4.2	2.7	1.9	1.4	1.2	0.4
	7.7	5.0	3.0	1.9	1.4	0.9	0.4
	7.1	4.4	2.8	2.2	1.3	0.9	0.7
Average value,	7.45	4.71	2.91	2.11	1.43	1.0	0.44

These average values are laid off as ordinates of a certain curve whose abscissas are the corresponding lengths of the bar. We can find an equation to satisfy these seven known points of the form: —

$$y = A\,x^6 + B\,x^5 + C\,x^4 + D\,x^3 + E\,x^2 + F\,x + G.$$

In drawing the curve I found that the sixth point would not come in the symmetrical curve, and so I threw it out; thus we shall have a curve of the fifth degree, the constants of whose equation can be determined from the following equations: —

$$y = A\,x^5 + B\,x^4 + C\,x^3 + D\,x^2 + E\,x + F \qquad \text{Eq. (1)}$$
$$F = 7.4.$$
$$4.7 = A + B + C + D + E + F.$$
$$2.9 = 32\,A + 16\,B + 8\,C + 4\,D + 2\,E + F.$$
$$2.1 = 243\,A + 81\,B + 27\,C + 9\,D + 3\,E + F.$$
$$1.4 = 3125\,A + 625\,B + 125\,C + 25\,D + 5\,E + F.$$
$$0.4 = 6561 \times 9\,A + 6561\,B + 729\,C + 81\,D + 9\,E + F.$$

Solving these equations, we find the coefficients $A\ B\ C\ D\ E\ F$, and substituting in Eq. (1), we get the equation of the first curve: —

$$y = -.000238\,x^5 + .00598\,x^4 - .0799\,x^3 + .701\,x^2 - 3.326\,x + 7.4.$$

The next step is to determine the second curve, or the rate of cooling. I took a piece of the bar used in the preceding experiment, and heated it in boiling water for twenty minutes. The rod was then taken out, wiped dry, placed horizontally across two wooden props, and the thermo-electric junction of copper and iron was placed upon the centre in a deep groove to insure a close connection. As the bar cooled the deflections on the galvanometer scale were read every minute. I performed the experiment four times, but the results were so nearly alike that I take only the last. The following numbers give the excesses of temperature above that of the air every minute: —

8.2		3.1		1.2		0.6	
	1.2		0.5		0.2		0.1
7.0		2.6		1.0		0.5	
	1.1		0.3		0.1		0.1
5.9		2.3		1.9		0.4	
	1.0		0.3		0.1		0.1
4.9		2.0		0.8		0.3	
	0.8		0.3		0.1		0.1
4.1		1.7		0.7		0.2	
	0.5		0.2		0.1		
3.6		1.5					
	0.4		0.3				

From these numbers we see that if any portion of the bar is at the temperature 8.2 it loses 1.2 units of temperature in one minute, etc. If these losses are laid off as ordinates right below the corresponding temperature, (as given by the ordinates of the first curve,) we shall have the second curve, whose area, for any definite portion, represents the loss of temperature per minute from a corresponding portion of the bar. If this second curve is drawn on co-ordinate paper, its area can be found by actual measurement, and we need not obtain its equation. The errors of such a method of measurement will be no greater than those which are liable to enter as experimental errors.

Making the measurement in the manner indicated above, and using the equation, $K = \frac{S}{A} \frac{\text{area}}{D_x t}$,

I found for point 2, $K = .2 \frac{1.97}{2.14} = .18.$

For point 3, $K = .2 \frac{1.38}{1.31} = .21.$

For point 4, $K = .2 \frac{1.05}{0.73} = .28.$

From these results it appears that the conductivity decreases as the temperature increases. It has also been shown, by Forbes, that the conductivity of iron decreases as the temperature increases.

The units in which the preceding conductivities are expressed are the centimeter, the minute, a centimeter deflection on the galvanometer scale, and the unit of heat is the amount of heat required to raise the temperature of 1 cm.^{3} of the glass one unit of deflection. To compare these conductivities with those determined by Forbes for iron, we must reduce them to functions of the units: the foot, the centigrade degree, and the minute.

S and $D_x t$ are ratios, and are independent of units. A contains cm.^{2} and "area" contains cm.^{3}

1 cm.^{2} = .155 sq. in. = .00107 sq. ft.
1 cm.^{3} = .061 cu. in. = .000035 cu. ft.

In determining the relation between one degree centigrade and one unit deflection, I made four measurements, using heated sand, the thermometer and thermopile being buried side by side.

5.5 cm.	corresponds	to a rise	in temp. of	12°.0
8.5	"	"	"	16.3
5.0	"	"	"	10.8
6.7	"	"	"	14.2

From these results, evidently,

1 cm.	corresponds	to a rise	in temp. of	2°.18
1	"	"	"	1.91
1	"	"	"	2.16
1	"	"	"	2.11
				4)8.36
Average				2.09

Hence, to reduce K to the required system of units, we divide by 2.09 and .00107 and multiply by .000035, or we multiply by .015.

The conductivities determined before become in the new system of units,

$$K = .00270.$$
$$K = .00315.$$
$$K = .00420.$$

These results can be summed up as follows. The numbers in the first row give, in centigrade degrees, the excess of the temperature of the bar above that of the air, and the second row the corresponding conductivities. The air was about 21° C.

Temperature of bar . .	19°.8	6°.08	4°.4
Conductivity	.00270	.00315	.00420

The conductivity of iron, as determined by Forbes, varied from .01337 at 0° C. to .00801 at 275° C. And the conductivity of sandstone, determined by burying thermometers from three to twenty-four feet below the surface, was found to be .000689.

The conductivity which I found for glass, therefore, lies between that of iron and that of sandstone. It also, like the conductivity of iron, increases as the temperature decreases. It has not been determined how the conductivity of sandstone varies with the temperature.

Conductivity of Sand.

To determine the conductivity of sand I used the following method. The sand is put in a thin metallic vessel; inside of this is put an exactly similar smaller metal vessel, which rests on the bottom of the outer one. Thus the sand forms a layer, protected on each side by the thin metal. These two vessels are entirely immersed in a larger vessel containing a known volume of water. The interior of the inner vessel is kept at a constant temperature (by means of steam). With a thermometer we can find the temperature of the outside water every minute, and thus construct a curve; the abscissas being the times and the ordinates the corresponding temperatures. In the next place, fill the outside vessel with the same volume of water used in the preceding experiment, heat it to a known temperature, and find the rate at which it cools. For this purpose we construct a second curve, having the times for abscissas and the temperatures for ordinates. Take any ordinate t, of the first curve, and let $d\,t_1 =$ the gain of temperature in time $d\,T$, then, if $V =$ the volume of the water, $d\,t_1 \,.\, V =$ the amount of heat gained in time $d\,T$. Take the same ordinate t, of the second curve, and let $d\,t_2 =$ the loss of temperature in time $d\,T$, then $d\,t_2 \,.\, V =$ the loss of heat from the water, when the temperature is t, in time $d\,T$. Hence the whole amount of heat that passes through the layer of sand in time $d\,T = Q = d\,t_1 \,.\, V + d\,t_2 \,.\, V$.

The formula for the quantity of heat that passes, in time T, through a section of thickness x, and area A, is $Q = K \,.\, A \,.\, \frac{t_0 - t}{x}\, T$. In the present case $T = d\,T$, and we have found the value for Q,

$$d\,t_1 \,.\, V + d\,t_2 \,.\, V = K \,.\, A \,.\, \frac{t_0 - t}{x}\, d\,T;$$

$$\therefore V\left[\frac{d\,t_1}{d\,T}+\frac{d\,t_2}{d\,T}\right]=K\,.\,A\,\frac{t_0-t}{x}\,.$$

At the limit we find the quantity of heat that passes through when the temperature of the water is t

$$=V(D_T\,t_1+D_T\,t_2)=K\,.\,A\,\frac{t_0-t}{x}\,.$$

From the first and second curve $D_T\,t_1$ and $D_T\,t_2$ are easily obtained, and as all the other quantities are known, K can be determined.

In my experiment I used for the outer vessel a tin can 7.1 cm. in height. This was covered with the exception of a hole in the top, around which was a collar. In this hole was placed a large test-tube of glass which just fitted the hole. The space between the test-tube and the walls of the can was filled with sand. This was placed in a larger tin can almost full of water, and suspended about 4 cm. from the bottom by a string. Steam was passed from a retort into the test-tube, and several times during the experiments I placed a thermometer in the test-tube, and it always showed that the temperature was 100° *C.*

At the start the water was 23.3°. I took down the temperatures every minute for almost an hour, but as I do not use these in constructing the curve, I give only the temperature determined every five minutes, after the first hour: —

Temperature.	Excess above 23°.3.	Temperature.	Excess above 23°.3.
29°.4	6°.1	34°.0	10°.7
30.1	6.8	34.3	11.0
30.7	7.4	34.6	11.3
31.3	8.0	34.9	11.6
31.8	8.5	35.1	11.8
32.3	9.0	35.3	12.0
32.8	9.5	35.5	12 2
33.3	10.0	35.7	12.4
33.6	10.3	35.9	12.6

The temperature of the air was 23.3°. The curve given by these points is not very regular; it changes several times from concave to convex. I therefore determined to take a small arc, find its equation, and investigate the conductivity as given by this arc. I considered only the points

7.4, 8.0, 8.5, 9.0, 9.5, 10.0.

If five small divisions of the co-ordinate paper are taken as one unit, an equation of the fifth degree, satisfying these six points, is found from: —

$$\begin{aligned}1.6 &= A + B + C + D + E + F. \qquad F = 1.48.\\ 1.7 &= 32\,A + 16\,B + 8\,C + 4\,D + 2\,E + F.\\ 1.8 &= 243\,A + 81\,B + 27\,C + 9\,D + 3\,E + F.\\ 1.9 &= 1024\,A + 256\,B + 64\,C + 16\,D + 4\,E + F.\\ 2.0 &= 3125\,A + 625\,B + 125\,C + 25\,D + 5\,E + F.\end{aligned}$$

From these equations $A\ B\ C\ D\ E\ F$ are found, and the required equation is:—

$$y = -0.0005\,x^4 + 0.006\,x^3 - 0.0245\,x^2 + 0.139\,x + 1.48.$$

Next, the same volume of water used in the preceding experiment was heated to 36°. The can was then placed on the same wooden props used before, and by means of the same thermometer the temperatures were read off every five minutes. The temperature of the air and of the water at starting was 23°. The following were the temperatures every five minutes:—

Temperature.	Excess above Air.	Temperature.	Excess above Air.
36°.0	13°.0	32°.1	9°.1
35.3	12.3	31.5	8.5
34.5	11.5	30.9	7.9
33.7	10.7	30.3	7.3
32.8	9.8		

I consider here only the temperatures included by the first equation; that is, from 9.8 to 7.3.

The equation of this second curve is found by the same process as in the first curve to be $y = 0.12\,x + 1.46$; therefore, the curve is a right line. The thickness of the layer of sand was 2.7 cm. $= x$. To determine A, I took the mean radius of the layer $= 1.35$ cm.

$\therefore A = 2\,\pi\,.\,1.35, 7.1 = 60.21$ cm.^{2}

h = height of the can = 7.1 cm.

V = volume of water. The water weighed 906 grammes; therefore, its volume was 906 cm.^{3}

$\frac{V}{A}\,x$ is constant, and = 40.62.

$$\therefore K = 40.62\,(D_T t_1 + D_T t_2)\,\frac{1}{t_0 - t}.$$

$$D_T t_1 = [D_x y] \text{ and } D_T t_2 \text{ always } = .12.$$

$$[D_x y]_{x=0} = 0.139; \qquad \therefore D_T t_1 + D_T t_2 = .259,$$

$$\text{and } t_0 - t = 100 - 30.7 = 69.3;$$

$$\therefore K = .1518.$$

$$[D_x y]_{x=1} = .106; \quad \therefore D_T t_1 + D_T t_2 = .226,$$
$$\text{and } t_0 - t = 100 - 31.3 = 68.7;$$
$$\therefore K = .1336.$$

$$[D_x y]_{x=2} = 0.097; \quad \therefore D_T t_1 + D_T t_2 = .217,$$
$$\text{and } t_0 - t = 100 - 31.8 = 68.2;$$
$$\therefore K = .1292.$$

$$[D_x y]_{x=3} = 0.100; \quad \therefore D_T t_1 + D_T t_2 = .22,$$
$$\text{and } t_0 - t = 100 - 32.3 = 67.7;$$
$$\therefore K = .132.$$

$$[D_x y]_{x=4} = .103; \quad \therefore D_T t_1 + D_T t_2 = .233,$$
$$\text{and } t_0 - t = 100 - 32.8 = 67.2;$$
$$\therefore K = .1348.$$

$$[D_x y]_{x=5} = .094; \quad \therefore D_T t_1 + D_T t_2 = .214,$$
$$\text{and } t_0 - t = 100 - 33.3 = 66.7;$$
$$\therefore K = .1303.$$

The average value of K is found, from the above observations, to be $K = .1353$.

The conductivities determined above should all be the same, and they are almost exactly the same, except the first; for after the first hour the sand must have reached a permanent state of heat, and the mean temperature was sensibly the same for all the values of K; the water only gained 2°.6 of temperature. Consequently, from these results it cannot be determined whether the conductivity of sand increases or decreases as the temperature increases. But to find how the conductivity varies with the temperature, the value of t_0 should be changed; therefore the temperature in the test-tube should be varied.

To reduce the average value of K to the same units used in glass, we multiply by .000035 and .0328 and divide by .00107, or we merely multiply by .00107 and find as the value of K, .000144771.

Forbes determined the conductivity of sand by burying thermometers below the surface (3 — 25 ft.), and found it to be .000169. The temperature of the sand for Forbes's conductivity was about 62° F. = 34°.4 C. The mean temperature of the sand in my experiment was about 50° C. From this it is probable that the conductivity of sand, like that of glass and of iron, decreases as the temperature increases. The specific gravity of the sand in my experiment was 16.

The errors arising from values of $D_x y$ are probably not very large. The arc of the first curve is almost a straight line, and the values of

$D_x y$ — after the first — were found to be almost the same. To obtain an absolutely reliable equation, it is necessary to take a great many points very near together; and as each point raises the equation one degree, and adds one more equation to those from which the coefficients are found, the mere mechanical work of finding the equation of the curve becomes very great.

No. XXV. — EFFECT OF THE TENSION OF MEMBRANES ON SOUND.

By G. F. Hartshorn.

Presented June 9, 1880.

Several experiments were tried bearing upon this subject by speaking alternately through two telephones, the mouth of one being covered with a membrane and the mouth of the other being open, the difference in sound between the two being observed by means of a telephone in the circuit. These experiments were not thought trustworthy, as it was difficult to speak with equal intensity and at precisely the same distances from the two instruments; to obviate these difficulties the following apparatus was devised.

An ordinary Bell telephone was provided with two tubes, which were placed opposite to each other on opposite sides of the vibrating plate. Rubber tubes were slipped upon these two tubes and led away to a mouth-piece which consisted of a chamber of clay, so moulded as to allow the extremities of the rubber tubes to terminate in broad glass funnels which were imbedded at a certain angle with each other in the clay. Another telephone was included in the circuit to enable one to hear what was transmitted.

Sounds sent to the telephones by both the mouth-pieces in the clay chamber would not be heard at the receiving telephone, because the equal vibrations striking opposite sides of the sending telephone plate would neutralize each other, but by closing alternately the connecting tubes the sound can be passed at will through either of the mouth-pieces.

In experimenting with the voice it was found that, with both a loose and a tight membrane, the articulation was plainer, and the sounds somewhat louder, than when passed through the open mouth-piece.